Problem Set #3

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Question 1

Fearon (2011) writes,

Thus, optimal play by a citizen must involve choosing whether to protest only according to whether her expectation of the probability the ruler will be deposed in that period is large enough to make protesting worth the risk. Namely, i should choose to protest in period t for sure ($P_{it} = 1$) *if* $E(G(M_t)) > s^* \equiv \frac{c}{c+b}$*, where the expectation is with respect to i's beliefs about other citizens' play at i's current information set.* $P_{it} = 0$ *if the inequality is reversed and any* $P_{it} \in [0, 1]$ *is optimal if* $E(G(M_t)) = s^*$*.* s^* *is the threshold probability of success such that individuals are just willing to turn out (p. 1669).*

Question 1a

Assume that in period t, one fourth of the citizens chooses to protest with probability 1 and the remaining three fourths choose to protest with probability 0. What then is the value of $E(G(M_t))$*?*

$E(G(M_t)) = .25$

Question 1b

Reconstruct the reasoning that leads to the conclusion that i should choose to protest in period t for sure if $E(G(M_t)) > s^*$

Because Fearon assumes that citizens cannot observe others' actions, each citizen must make their decision entirely on the basis of what they expect other citizens to do in the present period t. Since citizens realize payoffs in the case of successful revolutions ($b_t - D_{t-1}$) and suffer costs in the case of unsuccessful revolutions (c), they will only choose to protest where the expected benefits of doing so outweigh the costs (note that $D > b$, so this individual calculation is a choice between two net losses). The threshold probability of success s^* is defined as the cost of an unsuccessful protest divided by the sum of the cost of an unsuccessful protest and the benefit of a successful protest ($\frac{c}{c+b}$). This expression gives the net cost of protest in terms of the possible outcomes, and will be in the [0,1] interval. An individual will protest if and only if the expected probability of deposing the ruler $E(G(M_t))$ and thus realizing the benefit b is greater than the expected cost of protest, which is given by s^*.

Question 2

Consider the equilibrium described by Proposition 1 (p. 1671), in which the ruler chooses $g_t = \delta v$ *in every period* t*, and each citizen rebels in period* t *if and only if* $g_t < \delta v$*.*

Question 2a

What is the ruler's payoff in this equilibrium? (Note that his payoff is an infinite stream of discounted period payoffs). What is the ruler's payoff from choosing the best possible deviation from this strategy?

In equilibrium, the ruler is never removed. The ruler's payoff in equilibrium will be a stream of $v - \delta v$ discounted each period t, which we can express as:

$$\sum_{t=1}^{\infty} \delta^{t-1}(v - \delta v) = \frac{v - \delta v}{1 - \delta} = v \tag{1}$$

The best possible deviation to the ruler is to deviate from taking $g_t = 1 - v$ to taking $g_t = 0$ in $t = 1$ (intermediate positions would merely compromise the ruler's share in future periods without altering the probability of removal in the present period) If she does this, her payoff will be v. Because the payoff on the best deviation and on the equilibrium path are identical, the ruler has no incentive to deviate (this indifference result also indicates that the simple model has very little explanatory power because the equilibrium could be disrupted by an infinitesimal quantity).

Question 2b

Why is it the case that after any choice of g_t *by the ruler, no citizen has any incentive to deviate from the postulated strategy? Is the assumption that citizens benefit from higher values of* g_t *part of your explanation?*

A citizen's expected payoff from following the equilibrium strategy will be $\alpha\delta v$. Should they deviate in some period and choose to revolt, then $P_{it} = 1$. Assuming that other citizens continue to play their equilibrium strategy, $M_t = 0$ and thus $G(M_t) = 0$. The revolution will fail, and the revolting citizen's expected payoff will be their equilibrium payoff less the cost of an unsuccessful revolution, or $\alpha\delta v - c$. Because we know $c > 0$, the equilibrium payoff must be greater and no citizen has an incentive to deviate.

To verify the presence of a subgame-perfect equilibrium, we must also consider off-the-path strategies. There are two cases. As long as the ruler chooses $g_t > \delta v$, no other citizens will join our idiosyncratic citizen's revolt, and protesting only incurs a cost c. Where $g_t < \delta v$, all citizens will protest and $M_t = 1$, $G(M_t) = 1$. Citizens will realize a benefit b. Our citizen's overall payoff will be $\alpha g_t + b - D$. If she were to deviate, her payoff would be identical but without the "warm glow" benefit of participating in a successful revolution: $\alpha g_t - D$. Because $b > 0$, a citizen in this position has no incentive to deviate from the postulated strategy.

Question 2c

Is there an equilibrium in which each citizen chooses to rebel in period t after any g_t, and the ruler chooses $g_t = 0$ in every period? Explain.

Yes. This equilibrium can be sustained because no player has an incentive to deviate. As we have seen above, a revolting citizen's payoff will be $\alpha g_t + b - D$. If she were to deviate and not revolt, her payoff would be identical but without the "warm glow" benefit of participating in a successful revolution: $\alpha g_t - D$. Because $b > 0$, a citizen in this position has no incentive to deviate from the postulated strategy. Similarly, a ruler who will be deposed in $t = 1$ will (as shown above) choose $g_t = 0$. Any alternative allocation would still result in the ruler's deposition, so the ruler has no incentive to deviate from $g_t = 0$ because this is the maximum payoff available in t = 1 and the payoff stream in future rounds will sum to zero.

Question 3

Consider the version of the model with elections. What exactly is the function of elections in the equilibrium Fearon describes? How do they help to sustain an equilibrium in which the ruler chooses $g^* = \delta v$, which is better for the citizens than either of the equilibria (cases (1) and (2)) described by Proposition 2."

It appears that elections do *not* sustain a Pareto-optimal equilibrium. The ruler's choice of the governance outcome g has strategic implications for equilibrium behavior, but only as a coordinating device. Importantly, it plays no role in the citizens' payoff functions. Fearon has set up the model to give citizens a reason to desire successful revolutions no matter the underlying circumstances. In other words, citizens are indifferent to the value of g. This means there exist equilibria where the ruler actively hams the citizens. Fearon gestures in this direction with his discussion of elite pacts (1680), but the full implications of this aspect of his model do not appear in the text.

References

Fearon, James. 2011. "Self-Enforcing Democracy". *Quarterly Journal of Economics*, vol. 126, pp. 1661 - 1708.