Problem Set #2

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Question 1

Question 1a

When the players use the strategies Weingast specifies on p. 250, what are their payoffs?

In equilibrium, S will not transgress, A and B will acquiesce, and payoffs in the first round will be {2, 8, 8}. This equilibrium will be sustained indefinitely. We must then evaluate the infinite series of payoffs for each player. Assuming that players discount future payoffs (δ), we find that:

S will receive payoffs of:

$$2 + 2\delta + 2\delta^2 + 2\delta^3 + ... = \sum_{i=1}^{\infty} 2\delta^{i-1} = \frac{2}{1-\delta} \tag{1}$$

A and B will receive payoffs of:

$$8 + 8\delta + 8\delta^2 + 8\delta^3 + ... = \sum_{i=1}^{\infty} 8\delta^{i-1} = \frac{8}{1-\delta} \tag{2}$$

Question 1b

Suppose that S deviates from the postulated strategy and uses a strategy according to which S transgresses against B in some period 1, and then reverts to the postulated strategy in all subsequent periods. What would S's payoff be as a result of this deviation? Why do we know it is strictly lower than S's equilibrium payoff?

In the first period, S will transgress against B. Weingast's strategy profiles are given in terms of behavior in past rounds, so it is difficult to use them to determine behavior in the initial round of a game. Recall that A and B move simultaneously, so they are unable to base behavior in any particular round on their counterpart's actions. If A acquiesces but B challenges, payoffs in the first round will be {4, 9, 1}. S will receive payoffs of:

$$0 + 2\delta + 2\delta^2 + 2\delta^3 + ... = \sum_{i=1}^{\infty} 2\delta^{i} = \frac{2\delta}{1-\delta} \tag{3}$$

This will be strictly lower than S's equilibrium payoff as long as $\delta < 1$.

Question 1c

Is there any other alternative to A's equilibrium strategy that we should check in order to verify that A cannot profitably deviate from the equilibrium strategy in any subgame, given the strategies S and B use? Explain.

Assuming that this question takes up the postulated deviation by S in part 1b above, imagine that A were to acquiesce to S's transgression against B in the first round. Payoffs will be $\{4, 9, 1\}$. We know that in subsequent rounds S will transgress against both, and that A's best response to this will be to acquiesce (for a payoff of 2), and we also know that this equilibrium will be sustained thereafter with payoffs $\{8, 2, 2\}$ in subsequent periods. A would thus receive a payoff stream of:

$$9 + 2\delta + 2\delta^2 + 2\delta^3 + ... = 9 + \sum_{t=1}^{\infty} 2\delta^t = 9 + \frac{2\delta}{1-\delta} \tag{4}$$

If instead A were to follow the postulated equilibrium strategy and challenge following the transgression by S against B, payoffs would be $\{0, 7, 7\}$. A would thus receive a payoff stream of:

$$7 + 8\delta + 8\delta^2 + 8\delta^3 + ... = 7 + \sum_{t=1}^{\infty} 8\delta^t = 7 + \frac{8\delta}{1-\delta} \tag{5}$$

The payoff from pursuing the deviation will be strictly lower than the payoff from following the equilibrium strategy if and only if the following inequality is true:

$$7 + \frac{8\delta}{1-\delta} > 9 + \frac{2\delta}{1-\delta} \tag{6}$$

This inequality simplifies to:

$$\frac{6\delta}{1-\delta} > 2 \tag{7}$$

$$\delta > \frac{1}{4} \tag{8}$$

As a result, we see that A has no incentive to deviate in the subgame arising after S transgresses against B only for values of δ greater than $\frac{1}{4}$. We have examined the only deviation that offers A a higher payoff in the initial period (*i.e.* the only "interesting" deviation where the tradeoff will depend on the value of δ). The proposed equilibrium is thus subgame-perfect.

Question 2

Weingast introduces the second version of the model with the following comment:

"Model 1 allows no differentiation between the two groups, abstracting from the diversity of opinion about the appropriate boundaries of the state and hence about what actions constitute a transgression" (p. 248).

He describes one of the equilibria to the repeated game in similar terms:

"An important property of this equilibrium is that it supports a social consensus: All citizens hold the same views about transgressions and citizen duty. It thus respects a Lockean principle of active resistance to the sovereign in the face of transgressions" (p. 251).

Question 2a

Set aside Weingast's model for a moment. In general, is there a difference between opinions and views, on the one hand, and actions, choices, or strategies, on the other? How would you explain the difference? What is a consensus?

Yes. While actions, choices and strategies are ultimately based on opinions and views, in contexts of strategic interaction the correspondence is not straightforward. In these contexts, actions are a function of our own views and the anticipated actions of others. This means that we cannot infer beliefs (opinions and views) from actions in strategic contexts.

Question 2b

In Weingast's models, do the subjects have different views, or opinions, about anything? Explain.

It is difficult to say. Weingast certainly thinks so; he explicitly states that "these equilibria can be interpreted in terms of the implicit notion of citizen duty [or] ... passive obedience because citizens believe the sovereign rules by divine right" (249-250). But for the reasons given in 2a above, I am not so sure. Because Weingast's payoff structure is common knowledge, this seems to imply consensus on the very thing that he argues varies by individual in his model - the content of the idea of a "transgression against rights". So the model either assumes that we all agree on what counts as a transgression against rights, or (if the payoffs encode satisfaction from different states of the world) makes very strong assumptions about the extent to which we understand (have perfect information about) our peers' preferences and how they would react to the sovereign's actions. Payoffs are common knowledge, and they have to represent either a consensus on what transgressions are or consensus on how transgressions would be mediated through preferences. I think Weingast would opt for the latter interpretation, but since each group couldn't infer the preferences of other groups from those group's actions the assumption seems to be artificially driving the result.

Question 2c

Consider the passage from chapter 18 of Leviathan, where Hobbes writes, "Besides, if any..." Does the scenario Hobbes describes have any counterpart within Weingast's model?

The scenario Hobbes describes involves a disagreement among subjects (or between a subject and the sovereign) as to what constitutes a transgression by the sovereign. As discussed in 2b above, on what I take to be the more plausible interpretation of Weingast's model there exists disagreement across groups as to what constitutes a transgression, but the opinion of each group concerning potential transgressions is nevertheless common knowledge. As Hobbes points out, groups in this situation have no recourse but to return to the state of nature. Hobbes intends this passage as an argument against the derivation of a monarch's power from a social contract — it is an acknowledgment that sovereigns cannot credibly self-bind. Neither the pre-game social contract nor the post-game return to the state of nature by the losing group has a counterpart in Weingast's model.