Table 12.2: Matrix and Vector Transpose Properties

Inverse	$(A^T)^T = A$
Additive property	$(A+B)^T = A^T + B^T$
Multiplicative property	$(AB)^T = B^T A^T$
Scalar multiplication	$(cA)^T = cA^T$
Inverse transpose	$(A^{-1})^T = (A^T)^{-1}$
If A is symmetric	$A^T = A$

Table 12.3: Matrix Determinant Properties

Transpose property	$\det(A) = \det(A^T)$
Identity matrix	$\det(I) = 1$
Multiplicative property	$\det(AB) = \det(A)\det(B)$
Inverse property	$\det(A^{-1}) = \frac{1}{\det(A)}$
Scalar multiplication ($n \times n$)	$\det(cA) = c^n \det(A)$
If A is triangular or diagonal	$\det(A) = \prod_{i=1}^{n} a_{ii}$

Table 12.4: Matrix Inverse Properties

Inverse	$(A^{-1})^{-1} = A$
Multiplicative property	$(AB)^{-1} = B^{-1}A^{-1}$
Scalar multiplication ($n \times n$)	$(cA)^{-1} = c^{-1}A^{-1}$ if $c \neq 0$

Table 6.1: List of Rules of Differentiation

Sum rule	$(f(x)+g(x))' = f'(x) + g'(x)$
Difference rule	$(f(x)-g(x))' = f'(x) - g'(x)$
Multiply by constant rule	$f'(ax) = af'(x)$
Product rule	$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$
Quotient rule	$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$
Chain rule	$(g(f(x)))' = g'(f(x))f'(x)$
Inverse function rule	$(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}$
Constant rule	$(a)' = 0$
Power rule	$(x^n)' = nx^{n-1}$
Exponential rule 1	$(e^x)' = e^x$
Exponential rule 2	$(a^x)' = a^x(\ln(a))$
Logarithm rule 1	$(\ln(x))' = \frac{1}{x}$
Logarithm rule 2	$(\log_a(x))' = \frac{1}{x(\ln(a))}$
Trigonometric rules	$(\sin(x))' = \cos(x)$
	$(\cos(x))' = -\sin(x)$
	$(\tan(x))' = 1 + \tan^2(x)$
Piecewise rules	Treat each piece separately

Table 7.1: List of Rules of Integration

Fundamental theorem of calculus	$\int_a^b f(x)dx = F(b) - F(a)$
Rules for bounds	$\int_a^b f(x)dx = -\int_b^a f(x)dx$
	$\int_a^a f(x)dx = 0$
	$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ for $c \in [a,b]$
Linear rule	$\int (af(x) + bg(x))dx = a\int f(x)dx + b\int g(x)dx$
Integration by substitution	$\int_a^b f(g(u))g'(u)du = \int_{g(a)}^{g(b)} f(x)dx$
Integration by parts	$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$
Power rule 1	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$ if $n \neq -1$
Power rule 2	$\int x^{-1}dx = \ln\|x\| + C$
Exponential rule 1	$\int e^x dx = e^x + C$
Exponential rule 2	$\int a^x dx = \frac{a^x}{\ln(a)} + C$
Logarithm rule 1	$\int \ln(x)dx = x\ln(x) - x + C$
Logarithm rule 2	$\int \log_a(x)dx = \frac{x\ln(x)-x}{\ln(a)} + C$
Trigonometric rules	$\int \sin(x)dx = -\cos(x) + C$
	$\int \cos(x)dx = \sin(x) + C$
	$\int \tan(x)dx = -\ln(\|\cos(x)\|) + C$
Piecewise rules	Split definite integral into corresponding pieces

for any vector of dimension n, its length is given by $||\mathbf{a}|| = \sqrt{a_1^2 + a_2^2 + \ldots + a_n^2}$.

The **norm** of a vector $\mathbf{a}$, denoted $||\mathbf{a}||$, provides the length of the vector and can be related to the dot product according to[2] $||\mathbf{a}|| = \sqrt{\mathbf{a}\cdot\mathbf{a}}$. Thus the dot product allows us to easily compute the length of any vector. The same is true for the scalar difference between vectors: $||\mathbf{a}-\mathbf{b}|| = \sqrt{(\mathbf{a}-\mathbf{b})\cdot(\mathbf{a}-\mathbf{b})}$.

If we cannot write any vector in a set as a linear combination of the other vectors in the set, then we say the set of vectors is **linearly independent**.

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}.$$

The determinant of A is the difference of the diagonal products:

$$|A| = (a_{11}\cdot a_{22}) - (a_{12}\cdot a_{21}).$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix}.$$

Now each element in the top or bottom rows of the original matrix forms a full diagonal of three elements by going down or up, respectively, and to the right, as depicted in Figure 12.5.

The determinant will then equal the sum of the signed products of each diagonal, as follows:

$|A| = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{33}a_{21}a_{12}$.

to get the derivative we take the limit as $h \to 0$:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = f'(x) = \frac{d}{dx}f(x) = \frac{dy}{dx}.$$

partial derivative. It is written $\frac{\partial}{\partial x}f(x,y)$, or sometimes simply ∂_x, and means "treat every variable other than x as a constant, and just take the derivative with respect to x."

this gives us the chain rule:

$$\frac{dg(f(x))}{dx} = \frac{dg(u)}{du}\frac{du}{dx}, \text{ where } u = f(x).$$

$$\frac{d(f(x)g(x))}{dx} = \frac{df(x)}{dx}g(x) + f(x)\frac{dg(x)}{dx}.$$

In words, the **product rule** states that the derivative of the product is a sum of product terms. In each such term, one and only one of the constituent functions

To get the more general case of a^x for any a, we can use the properties of the exponential and logarithm to write it as $a^x = e^{x\ln(a)}$.[10] Next we use the chain rule. We let $g = e^x$ and $f = x\ln(a)$, so $\frac{da^x}{dx} = (e^{x\ln(a)})(\ln(a))$. Or, after rewriting the first term on the RHS one more time

$$\frac{da^x}{dx} = a^x(\ln(a)).$$

This is the **exponential rule.**

$y = g(f(x))$ and $y' = g'(f(x))f'(x) = (e^{x^2})(2x)$. This shows the nice thing about exponentials: they always return the original function, multiplied by the derivative of the term in the exponent. So the derivative of the really complicated function $y = e^{x^4-3x^2+1}$ is just $y' = (e^{x^4-3x^2+1})(4x^3 - 6x)$.

In other words, the definite integral of a function from a to b is equal to the antiderivative of that function evaluated at b minus the same evaluated at a.

We saw that the derivative is a linear operator in the previous chapter. This means that $(af + bg)' = af' + bg'$ for functions f and g and constants a and

More formally, to say that the integral is also a linear operator is to say that $\int(af(x) + bg(x))dx = a\int f(x)dx + b\int g(x)dx$.

As with substitution, we have presented integration by parts for the indefinite integral. It is not difficult to switch to the definite integral, though:

$$\int_a^b f(x)g'(x)dx = (f(x)g(x))|_a^b - \int_a^b f'(x)g(x)dx.$$