Poli 211 - Problem Set #1 4/3/20

1a) $N=\{1,\dots,n\}$ $X=\{(x_1,\dots,x_n)\in[0,1]^n \mid x_1+\dots+x_n=1\}$
(n is odd)

For each $i\in N$, $x \succsim_i y$ iff $x_i \ge y_i$

Core($\succsim$) = $\emptyset$ (QEF)

Two Approaches:

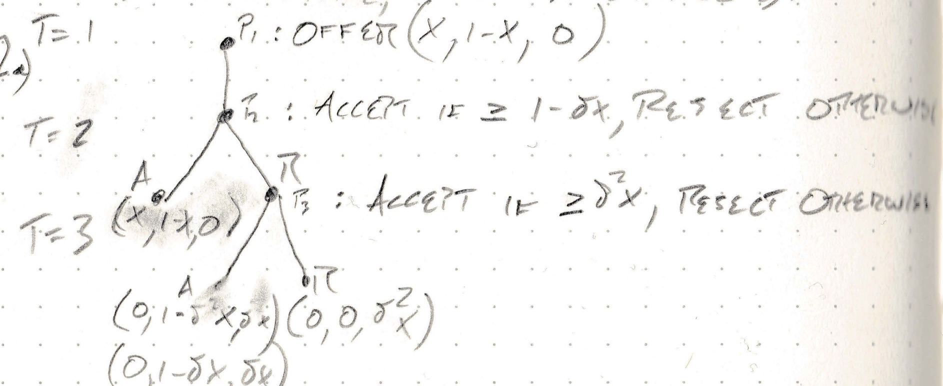
a) For any $x\in X$, subtract some ε from each n and for any y, add some ε to each n such that $(y_i \ge x_i) \ge \frac{n+1}{2}$ □

b) Choose any $x\in X$. Without loss of generality, assume $x_1>0$. Define y as $(0, x_2+\frac{x_1}{n-1},\dots, x_n+\frac{x_1}{n-1})$ $y\in X$. $y \succ_i x$ for all $i\ne 1$. Thus $y \succ x$ and there exists no $x\in X$ such that $x \succsim_m y$ for all $y\in X$. Thus, Core($\succsim$) $=\emptyset$ □

1b) $y=(\frac13,\frac13,\frac13)$ $x=(0,\delta,1-\delta)$

QEF: There are alternatives w, z such that $x \succ z \succ w \succ y$
Fix $0<\varepsilon<\delta$. Let $w := (\frac23-\varepsilon, \frac13+\varepsilon, 0)$ and $z := (1-\varepsilon, 0, \varepsilon)$. Then $x\succ z\succ w\succ y$. □
(Note that the majority preference relation is intransitive, implying an empty core).

2a)



If the game reaches T=3, Player 3 will propose to keep all the benefits $(0,0,\delta^2 x)$. Her decision rule will thus be to accept any offer $\ge \delta^2 x$. Knowing this, P_2 will offer $(0,1-\delta x,\delta x)$. Given this path of play, P_1 should offer a division of $(x, 1-x, 0)$. This unique SPE will be accepted in the first round.

2b) The continuation value $V_i(T,G)$ is the value to player i of continuing the game to round T and subgame G. We know that $V_1(3,G)=0$. Since P_3 will keep the whole surplus in round 3, $(0,0,\delta^2 x)$, her continuation value must be the difference between her round 3 and round 2 payoffs. $V_3(3,G)=\delta^2 x-0=\delta^2 x$. So we can conclude that $V_3(2,g)=\delta^2 x$. P_2 will likewise have a continuation value of $V_2(2,g)=(1-x)-(1-\delta x)$.

3a) This explanation relies on the "knife-edge" nature of indifference. To get the other players to vote for a distribution of benefits where they get nothing, a proposer would simply have to offer some infinitesimal epsilon. *

b) Because there is some chance that they will be recognized and thus able to keep all the benefits in the final period.

c) Because future benefits will be discounted.
*(And the other players won't have a best response)

4. $v_i = \frac{1}{n}$ (Prob. of recognition) $*$ $[1-\frac{n-1}{2}\delta v_i]$ (Benefits of a successful proposal) + $\frac{n-1}{n}$ (Prob. of non-recognition) $*$ $(\frac12)$ (Prob. of receiving a positive share) δv_i (Discounted continuation value). The final sentence gives the offers that will be made in the (unique) SPE.
A potential typo: the offer x_j made to $\frac{n-1}{2}$ other players is $\frac{\delta}{n}$ to $\frac{n-1}{2}$ others
(this is not stated)