Game Theory
Midterm Exam
11/4/18

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1a)

- Proposition 1 is the true theorem.
- Proposition 1 is of the form P -> Q.
  - We will assume P (s* is a Nash equilibrium strategy profile).
  - We would like to prove Q (s^*_i is rationalizable for each player i).
- Definitions:
  - A strategy profile is a Nash equilibrium if and only if each player is playing a best response to other players' strategies (with correct beliefs about the strategies other players will play).
  - A rationalizable strategy profile is a strategy that remains after iterated elimination of strategies that are never a best response.
    - Rationalizable strategy profiles are therefore best responses to at least some of the other players' strategies
- Proof: Assume P. Since we know that a Nash equilibrium s^*_i involves all players playing best responses, and we know that rationalizable strategy profiles are the best responses left over after iterated elimination of non-best responses, then we can conclude that s^*_i is rationalizable for each player i (Q). In fact, the set of Nash equilibria is a proper subset of the set of rationalizable strategies.

1b)

- However, this procedure does not work in the opposite direction (which is why Theorem 2 is untrue).
  - This is because a Nash equilibrium requires that *all* players be playing best responses, while the set of profiles that survive after iterated elimination of non-best responses lack this mutuality requirement. As a result, strategies may be unilaterally rationalizable without resulting in a Nash equilibrium. Additionally, a Nash equilibrium requires correct beliefs about the strategy that an opponent will play, while the surviving strategies after iterated elimination of non-best responses need not be paired with correct beliefs.

2a)

- Two conditions for a two-player normal-form game:
  - a) ∃ unique Nash equilibrium;
  - b) each player has at least two rationalizable strategies.
- Given these conditions, can either player have a strictly dominant strategy?

- A Nash equilibrium entails that all players are playing best responses to the strategy of every other player ($u_i(\sigma^*_i, \sigma^*_{-i}) > u_i(\sigma_i, \sigma^*_{-i})$ for all $i \in I$ and all $\sigma_i \in \Delta s_i$)
- A rationalizable strategy is one that survives iterated elimination of strategies that are never a best response.
- A strictly dominant strategy will always be played by player i regardless of the other players' strategies.
- Since each player has at least two rationalizable strategy profiles that survive iterated deletion of strategies that are never a best response, we know that neither player can have a strictly dominant strategy because the players will each have multiple best responses depending on the strategy profile (s_{-i}) chosen by the other players, which violates the definition of a strictly dominant strategy (above).
  - If a strategy is strictly dominated, it is never a best response to any strategy profile (Wiens 5.1, Tadelis 4.3).

2b)

1\2	H	T
H	-1, 1*	1*,-1
T	1*,-1	-1, 1*

2c)

Let p be the probability that player 1 plays H, and (1-p) be the probability that player 1 plays T.

Let q be the probability that player 1 plays H, and (1-q) be the probability that player 1 plays T.

(Pure strategies)

$BR_1(s_2) = \{ \{H\} \text{ if } S2 = T; \{T\} \text{ if } S2 = H \}$

$BR_2(s_1) = \{ \{T\} \text{ if } S1 = T; \{H\} \text{ if } S1 = H \}$

(Mixed – player 1 will mix when $u_1(H,p) = u_1(T,p)$; player 2 will mix when $u_2(H,q) = u_2(T,q)$)

$BR_1(q) = \{p = 0 \text{ if } q < ½; p \in [0,1] \text{ if } q = ½; p = 1 \text{ if } q > ½\}$

$BR_2(p) = \{q = 0 \text{ if } p > ½; q \in [0,1] \text{ if } p = ½; q = 1 \text{ if } p < ½\}$

$NE = \{ (p,q) = (½, ½) \}$

2d)

Set of rationalizable strategy profiles:

$S^2 = s_1 \times s_2$

$s_1 = \{ (H), (T), (p = ½) \}$

$s_2 = \{ (H), (T), (q = ½) \}$

The set of Nash equilibria (NE = $\{ (p,q) = (½, ½) \}$) is a proper subset of the set of rationalizable strategy profiles.

3a) See Figure 1

This is a game of imperfect information (h^c is not a singleton).

Information Sets:

A: $h^a = 1 = \{y\}$

B: $h^b = 2 = \{y^1\}, \{y^{11}\}$

C: $h^c = 1 = \{y^{111}, y^{1111}\}$

3b)

$S^3 = s_A \times s_B \times s_c$

$s_A = \{H, F\}$

$s_B = \{hh^1, hf^1, fh^1, ff^1\}$

$s_c = \{Y, N\}$

3c) (NE underlined)

	H (A)			F (A)	
B/C	Y	N	B/C	Y	N
hh^1	0*,0*,0*	0*,0*,0*	hh^1	0*,0*,0*	0*,0*,0*
hf^1	0,0*,0*	0*,0*,0*	hf^1	2*,0*,-1	-1,0*,2*
fh^1	-1,0*,2*	2*,0*,-1	fh^1	0*,0*,0*	0,0*,0*
ff^1	-1,0*,2*	2*,0*,-1	ff^1	2*,0*,-1	-1,0*,2*

3d) NE = { (H, hh^1, Y), (H, hh^1, N), (H, hf^1, N), (F, hh^1, Y), (F, hh^1, N), (F, fh^1, Y) }

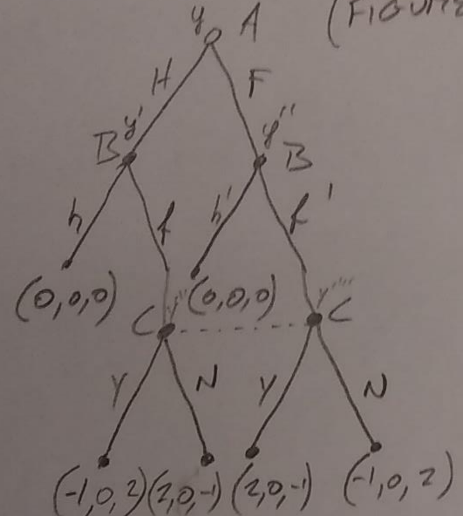
3e) See Figure 2.

A\C	Y	N
H	-1,2*	2*,-1
F	2*,-1	-1,2*

Pure Strategy NE = {$\emptyset$}

Mixed Strategy NE = { (p = ½), (q = ½) } (analogous to Matching Pennies)

3) $I = \{A, B, C\}$



(FIGURE 1)

a) IMPERFECT INFORMATION (h^c IS NOT A SINGLETON)

INFORMATION SETS:

A: $h^a = 1 = \{y\}$

B: $h^b = 2 = \{y'\}\{y''\}$

C: $h^c = 1 = \{y''', y''''\}$

b) $S_A = \{H, F\}$

$S_B = \{hh', hf', fh', ff'\}$

$S_C = \{Y, N\}$

$S^N = S_A \times S_B \times S_C$

c)

A: H

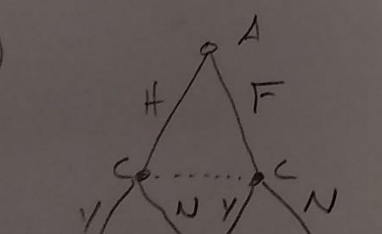
B\C	Y	N
hh'	0*,0*,0*	0*,0*,0*
hf'	0,0*,0*	0*,0*,0*
fh'	-1,0*,2*	2*,0*,-1
ff'	-1,0*,2*	2*,0*,-1

A: F

B\C	Y	N
hh'	0*,0*,0*	0*,0*,0*
hf'	2*,0*,-1	-1,0*,2*
fh'	0*,0*,0*	0,0*,0*
ff'	2*,0*,-1	-1,0*,2*

d) NE = {(H,hh',Y), (H,hh',N), (H,hf',N), (F,hh',Y) (F,hh',N), (F,fh',Y)}

e)



(FIGURE 2)

A\C	Y	N
H	-1,2*	2*,-1
F	2*,-1	-1,2*

$\Rightarrow$ NO NE